

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1247

Roll No.

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B.Tech.

(SEM. III) ODD SEMESTER THEORY

EXAMINATION 2013-14

DISCRETE STRUCTURES

Time : 3 Hours

Total Marks : 100

Note :—Attempt all Sections.

SECTION—A

1. Attempt all parts : (10×2=20)

(a) Let $A = \{a, \{a\}\}$. Determine whether the following statements are true or false :

(i) $\{a, \{a\}\} \in P(A)$

(ii) $\{a, \{a\}\} \subseteq P(A)$

(iii) $\{\{\{a\}\}\} \in P(A)$

(iv) $\{\{\{a\}\}\} \subseteq P(A)$.

(b) Find out the cardinality of the following sets :

$A = \{x : x \text{ is weeks in a leap year}\}$

$B = \{x : x \text{ is a +ve divisor of 24 and not equal to zero}\}$

$C = \{\{\{\}\}\}$

$D = \{\{\emptyset, \{\emptyset\}\}\}$.

- (c) How many symmetric and reflexive binary relations are possible on a set S with cardinality n ?
- (d) Define transitive closure with suitable example.
- (e) Find the minimum number of students in a class to show that five of them are born on same month.
- (f) Find the total number of squares in a chessboard.
- (g) Define Group with suitable example.
- (h) Define Lagrange's theorem. What is the use of the theorem ?
- (i) Determine by means of truth table the validity of DeMorgan's theorem for three variables :
 $(ABC)' = A' + B' + C'$
- (j) Define Binary Tree Traversal with example.

SECTION-B

2. Attempt all parts : (3×10=30)

- (a) Let (A, \leq) be a partially ordered set. Let \leq be a binary relation on A such that for a and b in A, a is related to b iff $b \leq a$.
 - (i) Show that \leq partially ordered relation.
 - (ii) Show that (A, \leq) is lattice or not.
- (b) (i) Define cyclic group with suitable example.
- (ii) Simplify the following Boolean functions using three variable maps :
 - (a) $F(x, y, z) = \Sigma(0, 1, 5, 7)$
 - (b) $F(x, y, z) = \Sigma(1, 2, 3, 6, 7)$

- (c) (i) Show that in a connected planar linear graph with 6 vertices and 12 edges, each of the regions is bounded by 3 edges.
- (ii) Show that a regular binary tree has an odd number of vertices.

SECTION-C

3. Attempt all parts : (5×10=50)

- (a) Let $A = \{2, 3, 6, 12, 24, 36\}$ and relation \leq be such that 'x \leq y' iff x divides y. Draw Hasse Diagram and find minimal and maximal elements.
- (b) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5, and 7.
- (c) Solve the following recurrence relation :
 - (i) $a_r - 7a_{r-1} + 10a_{r-2} = 0$, given that $a_0 = 0$ and $a_1 = 3$.
 - (ii) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 12$ satisfy the recurrence relation $a_r + C_1a_{r-1} + C_2a_{r-2} = 0$, determine a_r .

.OR

Prove by using mathematical induction that :

$$7 + 77 + 777 + \dots + 777\dots7 = 7/81[10^{n+1} - 9n - 10],$$

for every $n \in \mathbb{N}$.

- (d) (i) Given that the value of $P \rightarrow \bar{Q}$ is true, can you determine the value of $P \vee (P \leftarrow \rightarrow Q)$.
- (ii) Construct the truth table for the following statements :
 $(P \rightarrow \bar{Q}) \rightarrow \bar{P}$
 $P \leftarrow \rightarrow (\bar{P} \vee \bar{Q})$.

OR

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (4, 3), (2, 2), (2, 1), (3, 1)\}$ is a relation defined on A . Find Transitive closure of R using Warshall's algorithms.

- (e) (i) Suppose G is a finite cycle-tree graph with at least one edge. Show that G has at least two vertices of degree 1.
- (ii) Show that a connected graph with n vertices must have at least $(n - 1)$ edges.